

Birefringent Electroweak Textures

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The behaviour of electromagnetic waves propagating through an electroweak homilia string network is examined. This string network is topologically stable as a cosmic texture, and is characterized by the spatial variation of the isospin rotation of the Higgs field. As a consequence the photon field couples to the intermediate vector bosons, producing a finite range electromagnetic field. It is found that the propagation speed of the photon depends on its polarization vector, whence an homilia string network acts as a birefringent medium. We estimate the birefringent scale for this texture and show that it depends on the frequency of the electromagnetic wave and the length scale of the homilia string network.

PACS numbers: 98.80.Cq, 11.27.+d, 11.15.-q

I. INTRODUCTION

In the gauge theory of electroweak interactions the massless photon and the massive intermediate vector bosons, Z^0 and W^\pm , are unified within the group $SU(2) \times U(1)$ [1]. Recently we have shown that the electroweak model allows for the formation of a new kind of defect, called a homilia string [2]. Although locally string-like, this defect is homotopically classified as a texture. Since an homilia network behaves like a string locally, it is governed by the dynamics of a string network, thereby avoiding collapse associated with spherical texture defects (see for example [3]). Moreover, the local string-like geometry of homilia strings leads to a non-zero energy density that distinguishes homilia strings from the vacuum (i.e., $(D_\mu \Phi)^\dagger D^\mu \Phi$ and $Tr(F_{\mu\nu} F^{\mu\nu})$ are non-zero). As with textures and non-symmetric strings [4], homilia strings reconcile an undefined phase by forcing the Higgs field to undergo a rotation in isospin space, rather than forming a region of false vacuum.

The behaviour of photons in regions where the Higgs field varies spatially has been discussed previously [5–7]. Nambu [5] and Vachaspati [6] proposed definitions of the electromagnetic field tensor which lead to a finite range electromagnetic field within some topological defects (e.g., sphalerons [8]). However, a finite range photon field is considered to be unphysical in this context and Tornkvist [7] has proposed an ansatz intended to preserve the power-law behaviour of electromagnetic fields.

In this paper we examine the behaviour of electromagnetic waves propagating through an electroweak homilia string network. This network is characterized by the spatial variation of the orientation of the Higgs field in isospin space. The results reported here differ from previous work, since we analyse the behaviour of propagating electromagnetic waves when the definition of the vector bosons is endowed with an explicit spatial dependence. It is found that photons couple to massive intermediate vector bosons when the Higgs field rotates in isospin space. In this situation it is not possible to solve for propagating electromagnetic waves (photons) independently of the massive vector bosons, and all propagating solutions consist of a mixture of photons, Z^0 and W^\pm particles. Since photons propagate in the company of massive vector bosons, the phase velocity of electromagnetic waves is reduced. As a consequence cosmic textures, such as homilia string networks, have an effective refractive index which depends on the polarization state of the photon. These textures act like a birefringent medium and might be observable via measurements of cosmological anisotropy.

This paper is organised as follows. Section II describes the choice of gauge which is utilised to simplify the analysis of electroweak textures. However, the results are not dependent on the choice of gauge, and where relevant we demonstrate gauge invariance of the model predictions. In Sec. III the vector boson eigenvectors are calculated from a spatially dependent mass matrix. The coupling of the photon field to massive vector bosons is examined in Sec. IV. In Sec. V we solve for the propagating solutions of bosons fields using the coupled equations of motion. Finally, in Sec. VI we estimate the strength of the isospin coupling and the birefringence length scale using a homilia string network model. In appendix A we calculate the isospin coupling for an arbitrary choice of gauge and demonstrate gauge invariance of the model.

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II. HOMILIA STRINGS AND GAUGE TRANSFORMATIONS

Homilia strings were introduced in a earlier paper [2], where the orientation of the Higgs field in isospin space was chosen so that homilia strings separate into distinct components of the Higgs isodoublet. This enables us to define an α -string and a β -string within the context of the so called homilia gauge [2]. The homilia α -string is described in the homilia gauge as

$$\Phi_H^\alpha = \begin{pmatrix} |\phi^\alpha(r)|e^{i\theta} \\ |\phi^\beta(r)| \end{pmatrix}, \quad (1)$$

where $|\phi^\alpha(r)|$ and $|\phi^\beta(r)|$ are the real magnitudes of the scalar fields and the polar co-ordinate, θ , describes the non-trivial phase winding. The solutions and boundary conditions for $|\phi^\alpha(r)|$ and $|\phi^\beta(r)|$ are discussed in detail in reference [2].

To simplify the following discussion we perform a local gauge transformation

$$\Phi_H^\alpha \rightarrow \Phi^\alpha = e^{-i\tau^\alpha\theta} \Phi_H^\alpha = \begin{pmatrix} |\phi^\alpha(r)| \\ |\phi^\beta(r)| \end{pmatrix}, \quad (2)$$

where τ^α is the α -string generator

$$\tau^\alpha = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \quad (3)$$

The gauge transformation (2) results in a real Higgs field. It is important to emphasize that the line of undefined phase cannot be gauged away, since the gauge transformation is not defined at $r = 0$ (i.e., Eq. (2) requires $|\phi^\alpha(r=0)| = 0$, which preserves the undefined phase). Similarly a local gauge transformation can be performed for β -strings, using the β -string generator τ^β , where

$$\tau^\beta = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \quad (4)$$

We stress that the gauge transformation in Eq. (2) is not essential to the following discussion, but has been introduced for simplicity since it results in a real Higgs field. In appendix A we demonstrate that the results are valid for an arbitrary choice of gauge.

III. EIGENVECTORS OF THE MASS MATRIX

The Lagrangian density that describes the $SU(2) \times U(1)$ electroweak model is written as

$$\mathcal{L} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - \frac{1}{4} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) - \frac{1}{4} \lambda (\Phi^\dagger \Phi - \eta^2)^2, \quad (5)$$

where Φ is a complex isodoublet, $D_\mu = \partial_\mu - iqA_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + iq[A_\mu, A_\nu]$, $A_\mu = A_\mu^a \sigma_a$ and σ_a ($a = 1, 2, 3$) denotes the Pauli spin matrices. To include the $U(1)$ symmetry we have introduced the field A_μ^0 , which couples to the $U(1)$ generator via $\sigma_0 \equiv I \tan \theta_W$. The Lagrangian density (5) can be expanded to obtain

$$\begin{aligned} \mathcal{L} = & (\partial^\mu \Phi)^\dagger (\partial_\mu \Phi) + iqA_\mu^a [(\sigma^a \Phi)^\dagger \partial_\mu \Phi - (\partial_\mu \Phi)^\dagger \sigma^a \Phi] + q^2 A_\mu^a A_b^\mu \Phi^\dagger [\sigma_a, \sigma^b]_+ \Phi \\ & - \frac{1}{2} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + 2q\epsilon_{bc}^a A_\mu^b A_\nu^c) (\partial^\mu A_a^\nu - \partial^\nu A_a^\mu + 2q\epsilon_a^{bc} A_b^\mu A_c^\nu) \\ & - \frac{1}{4} \lambda (\Phi^\dagger \Phi - \eta^2)^2, \end{aligned} \quad (6)$$

where $[\sigma_a, \sigma_b]_+$ represents the anticommutation relation, $\sigma_a \sigma_b + \sigma_b \sigma_a$. Equation (6) describes the dynamics of the Higgs field Φ , the vector gauge fields A_μ^a and the interaction between the Higgs field and gauge fields. The equations of motion for the vector gauge fields are

$$\text{Tr}(\sigma^a{}^2) \left\{ \partial^\mu F_{\mu\nu}^a - \frac{q}{2} f_{bc}^a A^{b\mu} F_{\mu\nu}^c \right\} - i\frac{q}{2} \left\{ (D_\nu \Phi)^\dagger \sigma^a \Phi - (\sigma^a \Phi)^\dagger D_\nu \Phi \right\} = 0. \quad (7)$$

Equation (7) can be expanded to obtain

$$\begin{aligned} \square A_\nu^a - \partial_\nu \partial^\mu A_\mu^a = & -\frac{q^2}{4}(\Phi^\dagger[\sigma_b, \sigma^a]_+ \Phi) A_\nu^b - \frac{q}{2} f_{bc}^a A^{b\mu} \partial_\nu A_\mu^c - \frac{1}{2} f_{bc}^a A_\nu^c \partial^\mu A_\mu^b \\ & + \frac{q^2}{4} (f_{bc}^a f_{de}^c A^{b\mu} A_\mu^d A_\nu^e) + i \frac{q}{2} [(\partial_\nu \Phi)^\dagger \sigma^a \Phi - (\sigma^a \Phi)^\dagger \partial_\nu \Phi]. \end{aligned} \quad (8)$$

The last term in Eq. (8) is a current source term. Topological defects, such as homilia strings [2], textures and sphalerons [8], describe variations in the Higgs field which result in a non-zero current term in Eq. (8). For homilia strings (HS) the current term leads to static (non-trivial) vector boson fields described by

$$A_\theta^{HS} = \frac{n}{er} \begin{pmatrix} b(r) & c(r)e^{-in\theta} \\ c(r)e^{in\theta} & d(r) \end{pmatrix}. \quad (9)$$

Applying the gauge transformation (2) to Eq. (9) results in

$$A_\theta^{HS} = \frac{n}{er} \begin{pmatrix} b(r) - 1 & c(r) \\ c(r) & d(r) \end{pmatrix}. \quad (10)$$

The functions $b(r)$, $c(r)$ and $d(r)$ have been calculated numerically in reference [2]. Here we are interested in the implications arising from the spatial dependence of the vector boson eigenvectors, rather than the effects due to the current term. Consequently, in what follows we neglect the contribution from the current term.

Consider the mass matrix constructed from the $SU(2) \times U(1)$ Lagrangian density. From Eq. (6), the mass matrix term is

$$q^2 A_\mu^a A_\mu^b \Phi^\dagger [\sigma_a, \sigma^b]_+ \Phi. \quad (11)$$

We define

$$\mathcal{M}_{ab} = q^2 \Phi^\dagger [\sigma_a, \sigma_b]_+ \Phi, \quad (12)$$

which is written explicitly in matrix form as

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_{00} & \mathcal{M}_{01} & \mathcal{M}_{02} & \mathcal{M}_{03} \\ \mathcal{M}_{10} & \mathcal{M}_{11} & & \vdots \\ \mathcal{M}_{20} & & \ddots & \\ \mathcal{M}_{30} & \cdots & & \mathcal{M}_{33} \end{pmatrix} = q^2 \begin{pmatrix} |\Phi|^2 \tan^2 \theta_W & \chi^1(x) & \chi^2(x) & \chi^3(x) \\ \chi^1(x) & |\Phi|^2 & 0 & 0 \\ \chi^2(x) & 0 & |\Phi|^2 & 0 \\ \chi^3(x) & 0 & 0 & |\Phi|^2 \end{pmatrix}, \quad (13)$$

where θ_W is the Weinberg angle and $|\Phi|^2 = |\phi^\alpha|^2 + |\phi^\beta|^2$. The $\chi^a(x)$ -terms describe the isospin rotation of the Higgs field and are defined by

$$\chi^a(x) = \tan \theta_W \Phi^\dagger(x) \sigma^a \Phi(x) \quad (a = 1, 2, 3), \quad (14)$$

for which the following identity is valid

$$|\Phi|^4 \tan^2 \theta_W \equiv (\chi^1(x))^2 + (\chi^2(x))^2 + (\chi^3(x))^2. \quad (15)$$

Consider the situation where $|\Phi|$ is constant, but ϕ^α and ϕ^β vary spatially. The Higgs field is written as

$$\Phi(\mathbf{x}) = \begin{pmatrix} \phi^\alpha(\mathbf{x}) \\ \phi^\beta(\mathbf{x}) \end{pmatrix}. \quad (16)$$

To simplify the analysis we perform a gauge transformation (2). This results in a real Higgs field, with $\chi^2 = 0$. The Higgs field now becomes

$$\Phi(\mathbf{x}) = \begin{pmatrix} |\phi^\alpha(\mathbf{x})| \\ |\phi^\beta(\mathbf{x})| \end{pmatrix}. \quad (17)$$

The independent fields are determined from the mass matrix \mathcal{M} . The vector boson eigenvectors of the mass matrix (13) are (with $\chi^2 = 0$)

$$\mathbf{A}^\gamma(x) = \frac{\sin \theta_W}{|\Phi|^2 \tan \theta_W} \begin{pmatrix} -|\Phi|^2 \\ \chi^1(x) \\ 0 \\ \chi^3(x) \end{pmatrix} \quad (18a)$$

$$\mathbf{A}^Z(x) = \frac{\cos \theta_W}{|\Phi|^2 \tan \theta_W} \begin{pmatrix} |\Phi|^2 \tan^2 \theta_W \\ \chi^1(x) \\ 0 \\ \chi^3(x) \end{pmatrix} \quad (18b)$$

$$\mathbf{A}^W(x) = \frac{1}{|\Phi|^2 \tan \theta_W} \begin{pmatrix} 0 \\ \chi^3(x) \\ 0 \\ -\chi^1(x) \end{pmatrix} \quad (18c)$$

$$\mathbf{A}^{\overline{W}} = \begin{pmatrix} 0 \\ 0 \\ i \\ 0 \end{pmatrix}. \quad (18d)$$

Here A^γ and A^Z represent the photon and Z^0 eigenvectors, respectively. The vector bosons W^+ and W^- are described by the linear combinations

$$A^W = \frac{1}{2} (A^{W^+} + A^{W^-}) \quad (19a)$$

$$A^{\overline{W}} = \frac{1}{2} (A^{W^+} - A^{W^-}). \quad (19b)$$

W^+ and W^- are defined to be consistent with the standard decomposition of the gauge particles in the limit of the electroweak gauge [1,9,10]. Note that the eigenvectors (18) now depend on the space co-ordinate, which follows from the spatial dependence of the orientation of the Higgs field.

If we do not perform a local gauge transformation (2), the photon eigenvector becomes

$$\mathbf{A}^\gamma(x) = \frac{\sin \theta_W}{|\Phi|^2 \tan \theta_W} \begin{pmatrix} -|\Phi|^2 \\ \chi^1(x) \\ \chi^2(x) \\ \chi^3(x) \end{pmatrix}. \quad (20)$$

The photon field defined in Eq. (20) is equivalent to the definition in reference [6]. However, unlike the definitions utilised in references [5–7] we do not explicitly define the electromagnetic field tensor. Rather, we examine the behaviour of photons (20) in the context of the $SU(2) \times U(1)$ electroweak unification model. In Sec. V, we show that propagating solutions involve a mixture of vector bosons, and hence electromagnetic waves do not propagate independently of the massive vector bosons. The gauge invariance of the combined (propagating) state follows as a consequence of the gauge invariance of $Tr(F_{\mu\nu}F^{\mu\nu})$ (see reference [1]).

IV. ISOSPIN ROTATION AND VECTOR BOSONS

The spatial dependence of the photon eigenvector, $\mathbf{A}^\gamma(x)$, in Eq. (18a) is a consequence of Φ undergoing isospin rotation. This results in the photon field coupling to the intermediate vector bosons. To understand the consequences of this spatial dependence, we write the gauge fields in terms of the eigenvectors (18). Thus, for $\chi^2 = 0$

$$A_\mu^0 = -\cos \theta_W \alpha_\mu^\gamma + \sin \theta_W \alpha_\mu^Z \quad (21a)$$

$$A_\mu^1 = \frac{1}{|\Phi|^2 \tan \theta_W} (\sin \theta_W \chi^1 \alpha_\mu^\gamma + \cos \theta_W \chi^1 \alpha_\mu^Z + \chi^3 \alpha_\mu^W) \quad (21b)$$

$$A_\mu^2 = i \alpha_\mu^{\overline{W}} \quad (21c)$$

$$A_\mu^3 = \frac{1}{|\Phi|^2 \tan \theta_W} (\sin \theta_W \chi^3 \alpha_\mu^\gamma + \cos \theta_W \chi^3 \alpha_\mu^Z - \chi^1 \alpha_\mu^W). \quad (21d)$$

The fields α_μ^γ , α_μ^Z , α_μ^W and $\alpha_\mu^{\overline{W}}$ define the boson fields for an arbitrary isospin orientation of the Higgs field. Substituting Eqs. (21) into the Yang-Mills field tensor results in

$$\begin{aligned}
\mathcal{L}_{YM} &= Tr(F_{\mu\nu}F^{\mu\nu}) \\
&= (\partial_\mu\alpha_\nu^\gamma - \partial_\nu\alpha_\mu^\gamma)(\partial^\mu\alpha^{\gamma\nu} - \partial^\nu\alpha^{\gamma\mu}) + (\partial_\mu\alpha_\nu^Z - \partial_\nu\alpha_\mu^Z)(\partial^\mu\alpha^{Z\nu} - \partial^\nu\alpha^{Z\mu}) \\
&\quad + (\partial_\mu\alpha_\nu^W - \partial_\nu\alpha_\mu^W)(\partial^\mu\alpha^{W\nu} - \partial^\nu\alpha^{W\mu}) + (\partial_\mu\alpha_\nu^{\bar{W}} - \partial_\nu\alpha_\mu^{\bar{W}})(\partial^\mu\alpha^{\bar{W}\nu} - \partial^\nu\alpha^{\bar{W}\mu}) \\
&\quad + 2\rho^\mu\alpha^{\gamma\nu}\sin^2\theta_W(\rho_\mu\alpha_\nu^\gamma - \rho_\nu\alpha_\mu^\gamma) + 2\rho^\mu\alpha^{Z\nu}\cos^2\theta_W(\rho_\mu\alpha_\nu^Z - \rho_\nu\alpha_\mu^Z) \\
&\quad + 2\rho^\mu\alpha^{W\nu}(\rho_\mu\alpha_\nu^W - \rho_\nu\alpha_\mu^W) + 2\rho^\mu\alpha^{\gamma\nu}\sin\theta_W\cos\theta_W(\rho_\mu\alpha_\nu^Z - \rho_\nu\alpha_\mu^Z) \\
&\quad + 2\rho^\mu\alpha^{\gamma\nu}\sin\theta_W(\partial_\mu\alpha_\nu^W - \partial_\nu\alpha_\mu^W) + 2\rho^\mu\alpha^{Z\nu}\cos\theta_W(\partial_\mu\alpha_\nu^W - \partial_\nu\alpha_\mu^W) \\
&\quad - 2\rho^\mu\alpha^{W\nu}\sin\theta_W(\partial_\mu\alpha_\nu^\gamma - \partial_\nu\alpha_\mu^\gamma) - 2\rho^\mu\alpha^{W\nu}\cos\theta_W(\partial_\mu\alpha_\nu^Z - \partial_\nu\alpha_\mu^Z) \\
&\quad + \mathcal{L}_{int}(q, q^2) + \mathcal{L}_{int+}(q\rho_\mu),
\end{aligned} \tag{22}$$

where $\mathcal{L}_{int}(q, q^2)$ represents terms involving the structure constants (see reference [1]), and $\mathcal{L}_{int+}(q\rho_\mu)$ includes terms involving a mixture of q and the strength of the isospin rotation, ρ_μ , i.e.,

$$\begin{aligned}
\mathcal{L}_{int+}(q\rho_\mu) &= i2q\alpha^{\gamma\nu}\rho^\mu\sin^2\theta_W(\alpha_\mu^{\bar{W}}\alpha_\nu^\gamma - \alpha_\mu^\gamma\alpha_\nu^{\bar{W}}) \\
&\quad + i2q\alpha^{\gamma\nu}\rho^\mu\sin\theta_W\cos\theta_W(\alpha_\mu^{\bar{W}}\alpha_\nu^Z - \alpha_\mu^Z\alpha_\nu^{\bar{W}}) \\
&\quad + i2q\alpha^{Z\nu}\rho^\mu\cos^2\theta_W(\alpha_\mu^{\bar{W}}\alpha_\nu^Z - \alpha_\mu^Z\alpha_\nu^{\bar{W}}) \\
&\quad + i2q\alpha^{Z\nu}\rho^\mu\sin\theta_W\cos\theta_W(\alpha_\mu^{\bar{W}}\alpha_\nu^\gamma - \alpha_\mu^\gamma\alpha_\nu^{\bar{W}}) \\
&\quad + i2q\alpha^{W\nu}\rho^\mu(\alpha_\mu^{\bar{W}}\alpha_\nu^W - \alpha_\mu^W\alpha_\nu^{\bar{W}}).
\end{aligned} \tag{23}$$

The first four terms in Eq. (22) are the usual field strength terms. However, the remaining terms are a consequence of the spatial dependence of $\chi^a(x)$. The strength of the isospin rotation, ρ_μ , is defined as

$$\rho_\nu = \frac{(\partial_\nu\chi^1)\chi^3 - (\partial_\nu\chi^3)\chi^1}{|\Phi|^4\tan^2\theta_W} \quad (\chi^2 = 0). \tag{24}$$

From Eq. (24) we have the relation

$$\rho_\mu\rho^\mu \equiv \frac{(\partial_\mu\chi^1)(\partial^\mu\chi^1) + (\partial_\mu\chi^3)(\partial^\mu\chi^3)}{|\Phi|^4\tan^2\theta_W}. \tag{25}$$

The quantity ρ_μ represents the fundamental coupling strength, since we can write

$$(\sin\theta_W)^{-1}\partial_\mu\mathbf{A}^\gamma = (\cos\theta_W)^{-1}\partial_\mu\mathbf{A}^Z = \rho_\mu\mathbf{A}^W \tag{26a}$$

$$\partial_\mu\mathbf{A}^W = -\rho_\mu(\mathbf{A}^\gamma\sin\theta_W + \mathbf{A}^Z\cos\theta_W). \tag{26b}$$

Therefore ρ_μ defines the relationship between the isospin rotation and the derivatives of the eigenvectors of the vector bosons. From Eqs. (26) we see that the field strength term, $Tr(F_{\mu\nu}F^{\mu\nu})$, leads immediately to isospin coupling, as a direct consequence of the spatial dependence of the eigenvectors.

From Eq. (22) we note that the rotation of the Higgs field produces a term of the form

$$2\rho^\mu\alpha^{\gamma\nu}\sin^2\theta_W(\rho_\mu\alpha_\nu^\gamma - \rho_\nu\alpha_\mu^\gamma). \tag{27}$$

If we define the isospin rotation direction to be in the z -direction, such that ρ_z is the only non-zero component of ρ_μ , then Eq. (27) becomes

$$2\rho_z\rho^z\sin^2\theta_W(\alpha_\nu^\gamma\alpha^{\gamma\nu} - \alpha_z^\gamma\alpha^{\gamma z}). \tag{28}$$

When $\nu \neq z$, Eq. (28) represents a pseudo-mass term for the photon (i.e., when the polarization state is perpendicular to the z -direction). We define the photon pseudo-mass, P_γ , by

$$P_\gamma^2 = \rho_\mu\rho^\mu\sin^2\theta_W. \tag{29}$$

It is apparent that the rotation of the Higgs field (in isospin space) results in the photon coupling to the Higgs field. Coupling of the photon field to the massive intermediate vector bosons prevents the photon propagating as a massless particle. However, the photon rest mass is determined by the eigenvalue of the mass matrix and is always zero. If the photon polarization state is parallel to the isospin rotation (i.e., the z -direction), the photon decouples from the massive intermediate vector bosons and does indeed propagate as a massless particle. As a consequence of this polarisation dependence an homilia string network (texture) acts as a birefringent medium. In Sec. V we examine the consequences of this observation in more detail.

V. CONSEQUENCES OF ISOSPIN ROTATION

To simplify the analysis of the photon's behaviour in a region of isospin rotation, we approximate the isospin rotation by an average value

$$\langle \rho_\mu \rho^\mu \rangle = \rho^2, \quad (30)$$

where ρ is a constant. A co-ordinate system is chosen so that the isospin rotation is in the z -direction

$$\rho_\nu = \begin{cases} \rho & \nu = z \\ 0 & \nu \neq z. \end{cases} \quad (31)$$

Adopting the simplification (30), the equations of motion become

$$\begin{aligned} \partial^\mu \partial_\mu \alpha_\nu^\gamma - \partial_\nu \partial^\mu \alpha_\mu^\gamma - \sin^2 \theta_W [(1 - \delta_{\nu z}) \rho^2 (\alpha_\nu^\gamma + \alpha_\nu^Z) + (2 - \delta_{\nu z}) \rho \partial_z \alpha_\nu^W] \\ + \sin^2 \theta_W (\rho \delta_{\nu z} \partial^\mu \alpha_\mu^W) + \mathcal{O}(q) = 0 \end{aligned} \quad (32a)$$

$$\begin{aligned} \partial^\mu \partial_\mu \alpha_\nu^Z - \partial_\nu \partial^\mu \alpha_\mu^Z - M_Z^2 \alpha_\nu^Z - \cos^2 \theta_W [(1 - \delta_{\nu z}) \rho^2 (\alpha_\nu^\gamma + \alpha_\nu^Z) \\ - \cos^2 \theta_W [(2 - \delta_{\nu z}) \rho \partial_z \alpha_\nu^W - \rho \delta_{\nu z} \partial^\mu \alpha_\mu^W] + \mathcal{O}(q) = 0 \end{aligned} \quad (32b)$$

$$\begin{aligned} \partial^\mu \partial_\mu \alpha_\nu^W - \partial_\nu \partial^\mu \alpha_\mu^W - M_W^2 \alpha_\nu^W - [(1 - \delta_{\nu z}) \rho^2 \alpha_\nu^W + (2 - \delta_{\nu z}) \rho \partial_z (\alpha_\nu^\gamma + \alpha_\nu^Z) \\ + [\rho \delta_{\nu z} \partial^\mu (\alpha_\mu^\gamma + \alpha_\mu^Z)] + \mathcal{O}(q) = 0 \end{aligned} \quad (32c)$$

$$\partial^\mu \partial_\mu \alpha_\nu^{\overline{W}} - \partial_\nu \partial^\mu \alpha_\mu^{\overline{W}} - M_W^2 \alpha_\nu^{\overline{W}} + \mathcal{O}(q) = 0, \quad (32d)$$

where $\mathcal{O}(q)$ denotes higher order interaction terms, and M_Z and M_W are the masses of the Z^0 and W^\pm particles, respectively, i.e.,

$$M_Z^2 = q^2 [1 + \tan^2 \theta_W] |\Phi|^2 \quad (33a)$$

$$M_W^2 = q^2 |\Phi|^2. \quad (33b)$$

When analysing the behaviour of the photon, $\alpha^{\overline{W}}(x)$ is set to zero. We adopt the following ansatz for $\alpha_\nu^\gamma(t, \mathbf{x})$, $\alpha_\nu^Z(t, \mathbf{x})$ and $\alpha_\nu^W(t, \mathbf{x})$:

$$\alpha_\nu^\gamma(t, \mathbf{x}) = a_\nu e^{i(\mathbf{k} \cdot \mathbf{x} \pm \omega t)} \quad (34a)$$

$$\alpha_\nu^Z(t, \mathbf{x}) = b_\nu e^{i(\mathbf{k} \cdot \mathbf{x} \pm \omega t)} \quad (34b)$$

$$\alpha_\nu^W(t, \mathbf{x}) = c_\nu e^{i(\mathbf{k} \cdot \mathbf{x} \pm \omega t)}. \quad (34c)$$

where \mathbf{k} and ω are constants, and a_ν , b_ν and c_ν are polarization vectors. For a linearly polarized wave, described by Eqs. (34), the interaction terms \mathcal{L}_{int} and \mathcal{L}_{int+} cancel, simplifying the equations of motion. Substituting Eqs. (34) into the equations of motion (32) and solving for k results in the following polynomial in ω :

$$\begin{aligned} & \{ [\omega^2 - k^2 - \rho^2 \sin^2 \theta_W (1 - \delta_{\nu z})] \\ & \times [\omega^2 - k^2 - M_W^2 - \rho^2 (1 - \delta_{\nu z})] + 4\rho^2 k_z^2 (1 - \delta_{\nu z}) \} \\ & \times \{ [\omega^2 - k^2 - M_Z^2 - \rho^2 \cos^2 \theta_W (1 - \delta_{\nu z})] \\ & \times [\omega^2 - k^2 - M_W^2 - \rho^2 (1 - \delta_{\nu z})] + 4\rho^2 k_z^2 (1 - \delta_{\nu z}) \} \\ & = \{ \rho^2 \sin^2 \theta_W [\omega^2 - k^2 - M_W^2 - \rho^2 (1 - \delta_{\nu z})] + 4\rho^2 k_z^2 (1 - \delta_{\nu z}) \} \\ & \times \{ \rho^2 \cos^2 \theta_W [\omega^2 - k^2 - M_W^2 - \rho^2 (1 - \delta_{\nu z})] + 4\rho^2 k_z^2 (1 - \delta_{\nu z}) \}. \end{aligned} \quad (35)$$

Here $k = |\mathbf{k}|$ and k_z denotes the component of \mathbf{k} in the z -direction. Equation (35) governs the relationship between k and ω and hence determines the propagation speed of the vector bosons. Note that $\delta_{\nu z} \neq 0$ when the polarization direction of the gauge bosons has a component in the direction of isospin rotation; hence different polarization states propagate at different velocities.

The equations governing the field intensities can also be written in terms of the photon field intensity a_ν , i.e.,

$$\begin{aligned} b_\nu \{ \omega^2 - k^2 - \rho^2 [(\cos \theta_W)^2 - \sin^2(\theta_W)^2] - M_Z^2 \} \\ = a_\nu \{ \omega^2 - k^2 - \rho^2 [(\sin \theta_W)^2 - (\cos \theta_W)^2] \} \end{aligned} \quad (36a)$$

$$c_\nu (\omega^2 - k^2 - \rho^2 - M_W^2) = (a_\nu + b_\nu) (2ik_z \rho). \quad (36b)$$

The α_ν^W -field decouples from the photon and Z^0 fields when there is no component of k in the z -direction (i.e., $k_z = 0$). However, for $k_z \neq 0$ the α_ν^W -field is coupled to, and 90° out of phase with the photon and Z^0 fields. This arises because of the imaginary term in Eq. (36b).

When the isospin rotation strength is zero ($\rho = 0$), Eq. (35) simplifies to

$$(\omega^2 - k^2)(\omega^2 - k^2 - M_W^2)(\omega^2 - k^2 - M_Z^2) = 0. \quad (37)$$

The solutions to Eq. (37) correspond to the massless photon ($\omega = \pm k$) and massive Z^0 and W^\pm intermediate vector bosons ($E^2 - p^2 = m^2$). For $\rho = 0$, the solutions to Eq. (37) separate into the distinct fields, α_ν^γ , α_ν^Z and α_ν^W . This is because in this case the solutions for a_ν , b_ν and c_ν decouple in Eq. (36). Hence each vector boson field can be determined independently of the others. However, when $\rho \neq 0$, the vector bosons are coupled and it is no longer possible to find independent solutions for the individual fields (i.e., a_ν , b_ν and c_ν are all non-zero for a given solution). There are still three solutions corresponding to three propagating states, however, each of these three solutions consist of a mixture of intermediate vector bosons. As ρ is reduced to zero, these mixed solutions deform continuously into independent solutions for the spin-1 fields. Consequently, for small isospin rotation, we can interpret the mixed particle solutions as describing predominantly photons, Z^0 and W^\pm particles.

For the case $\rho \ll k \ll |\Phi|$, the photon solution to Eq. (35) may be written as

$$\omega^2 = k^2 + \rho^2 \sin^2 \theta_W (1 - \delta_{\nu z}) + \mathcal{O}\left(\frac{\rho^2 k_z^2}{M_W^2}\right) + \mathcal{O}\left(\frac{\rho^4}{|\Phi|^2}\right). \quad (38)$$

Equation (38) implies that the photon propagates as if it had a pseudo-mass P_γ , whence Eq. (29) becomes

$$P_\gamma \approx \rho \sin \theta_W. \quad (39)$$

However, the photon has zero rest mass since its mass matrix eigenvalue is always zero; nevertheless, it propagates with a phase velocity less than the speed of light for a constant Higgs field. When the isospin rotation strength, ρ , is small we can model the behaviour of the photon using an effective refractive index n_H . For $\rho = \text{const}$ we find (for $\omega \gg P_\gamma$)

$$n_H \equiv \frac{k}{\omega} \approx \sqrt{1 - \frac{P_\gamma^2}{\omega^2}} \approx 1 - \frac{1}{2} \frac{P_\gamma^2}{\omega^2} + \mathcal{O}(P_\gamma^4/\omega^4). \quad (40)$$

The mixed particle solution, arising from the isospin coupling, only propagates with $k < \omega$; this is due to the admixture of massive and massless spin-1 bosons. When $\omega = 0$, the equations describe a finite range electromagnetic field, where $k = \pm i P_\gamma$.

The refractive index (40) depends on whether the polarization direction is parallel, or perpendicular to the rotation of the Higgs field in isospin space. Hence we can write

$$n_\perp \approx 1 - \frac{1}{2} \frac{P_\gamma^2}{\omega^2} + \mathcal{O}(P_\gamma^4/\omega^4) \quad (41a)$$

$$n_\parallel = 1. \quad (41b)$$

Therefore a region where the isospin orientation of the Higgs field varies spatially acts as a birefringent medium [11]. The relative phase shift, $\Delta\varphi$, between orthogonal polarization states propagating through the birefringent medium is given by

$$\Delta\varphi = kd|n_\perp - n_\parallel|, \quad (42)$$

where to first order the birefringence length scale is ($k \gg P_\gamma$)

$$d \approx \frac{4\pi k}{P_\gamma^2}. \quad (43)$$

Equation (42) can be generalised to

$$\Delta\varphi = dP_\gamma^2 \sin \theta / 2k, \quad (44)$$

where θ is the angle between the direction of propagation and the vector ρ_μ . The angular dependence of the phase shift arises because an electromagnetic wave propagating parallel to ρ_μ has both polarization vectors perpendicular to ρ_μ (i.e., no relative phase shift); compared to a wave propagating perpendicular to ρ_μ , in which one polarization state is parallel, and the other perpendicular to ρ_μ (i.e., a phase shift of $\Delta\varphi = dP_\gamma^2/2k$).

VI. COUPLING STRENGTH FOR ISOSPIN ROTATION

To calculate the size of P_γ and d we require an estimate of the isospin rotation of the Higgs field. We can use a homilia string network [2] to estimate ρ_μ , since this defect is stable in the electroweak model. However, birefringent properties arise for any defect which predicts an isospin rotation of the Higgs field.

Homilia strings induce an isospin rotation at all points in the Universe, since the functions, $|\phi^\alpha(r)|$ and $|\phi^\beta(r)|$, never limit to a constant value (see e.g., the vortex solution in reference [2]). This behaviour reflects the texture nature of the homilia string network. In [2] we obtained an approximate vortex solution for a homilia α -string based on cylindrical symmetry. We describe the homilia string network by a single length scale, $L(t)$, in the same manner as cosmic string networks are characterized [12]. Here $L(t)$ is the average distance between a segment of homilia string and its nearest neighbours. The vortex solution is invariant under rescaling, $\mathbf{x} \rightarrow \mathbf{x}' = \mathbf{x}/L(t)$. Hence we write the homilia α -string for an arbitrary length scale $L(t)$ as

$$\Phi(r') = \begin{pmatrix} |\phi^\alpha(r')|e^{i\theta} \\ |\phi^\beta(r')| \end{pmatrix}. \quad (45)$$

This vortex solution is scale invariant, since for $L \gg (\sqrt{\lambda}\eta)^{-1}$ we find $|\Phi| \approx \text{const}$. Note that in general $|\Phi| \neq \text{const}$ for the homilia string vortex solution since there must always be a deviation from η at $r = 0$ [2]. However, the size of this deviation depends on the separation distance and for large separations it is a good approximation to write $|\Phi| \approx \text{const}$. The energy density of the homilia string network increases as we reduce L and hence the invariance of the vortex solution under rescaling by L does not result in the collapse of the network.

Using Eq. (45) we find

$$P_\gamma(t) = \sin \theta_W (\rho_\mu(t) \rho^\mu(t))^{1/2} \approx \sin \theta_W \frac{(\hat{\rho}_\mu \hat{\rho}^\mu)^{1/2}}{L(t)}, \quad (46)$$

where $\hat{\rho}_\mu \hat{\rho}^\mu$ is a numerically determined quantity that describes the strength of the isospin rotation of the homilia string. From a cylindrically symmetric $SU(2) \times U(1)$ homilia string [2], we obtain an average value of $\langle \hat{\rho}_\mu \hat{\rho}^\mu \rangle^{1/2} \approx 1.3$, with $L(t) = 1\text{GeV}^{-1}$. Equation (46) describes the relationship between the pseudo-mass of the photon and the length scale of the homilia string network.

The quantity $L(t)$ is a measure of the length scale of the network and can be estimated from analytical models of string network evolution [12]. To take into account the two types of electroweak homilia strings (HS) we write

$$L(t) = L_{HS}(t) \approx \frac{1}{\sqrt{N}} L_{CS}(t), \quad (47)$$

where N is the order of the symmetry breaking group, $U(N)$, and L_{CS} denotes the length scale for a cosmic string network. For the electroweak model $N = 2$. Consider a flat Universe with a zero cosmological constant. $L(t)$ can be approximated in the matter dominated era by [12]

$$L(t) = \left(\frac{9k_m(k_m + c_m)}{8N} \right)^{1/2} t, \quad (48)$$

where $k_m = 0.49$ describes the small scale structure and $c_m = 0.17$ is the loop chopping efficiency [12]. For electroweak homilia strings we find $L(t) \approx 0.43t$, hence the length scale of the photon pseudo-mass is of the order of the size of the observable Universe.

Although the photon pseudo-mass is very small, it can be differentiated from quantum fluctuations since the former is coherent over the length scale $L(t)$. Because the length scale is $L(t) \approx 0.43t$, the direction of ρ_μ is essentially constant across the observable Universe. As a consequence the birefringence scale exhibits an angular dependence described by $d \propto k/(\sin \theta P_\gamma^2)$, where θ is the angle between the direction of propagation and the vector ρ_μ .

Isospin rotation can in principle be measured by examining the orientation of polarized synchrotron radiation emitted from distant active galactic nuclei. Using the estimate from Eq. (43) we find the birefringence length scale of the homilia network to be

$$d \approx \frac{\omega(0.43t)^2}{(\hat{\rho} \sin \theta_W)^2}. \quad (49)$$

The typical frequency of synchrotron radiation from distant galaxies is $\omega \sim 10^{-16}$ GeV and hence in the present epoch we obtain an order of magnitude estimate of $d \sim 10^{65}$ GeV $^{-1}$, which is 10^{24} times the size of the observable Universe.

Nodland and Ralston [13] claim to have detected a birefringence length scale of the order of the size of the observable Universe. Therefore, the isospin rotation of the Higgs field due to the presence of a homilia string network does not explain their observation [13]. Although we could conjecture a smaller network length scale, this would not explain why the orientation of the isospin rotation of the Higgs field is coherent over ten billion lightyears.

For the model to be in agreement with the results of Nodland and Ralston [13], we require the existence of an electroweak texture which possesses a ρ_μ -direction which is coherent across the observable Universe and induces an average pseudo-mass of $\langle P_\gamma \rangle \approx 10^{-28} \text{GeV}^{-1} = 10^{-52} \text{ g}$. The average pseudo-mass of 10^{-52} g is two orders of magnitude smaller than the upper limit of the ‘photon mass’ as measured by Lakes [14]. It is conceivable that the existence of such a texture could be verified in the near future.

VII. CONCLUSIONS

In this paper we have examined the behaviour of the photon field in an electroweak homilia string network. It is found that the spatial dependence of the orientation of the Higgs field in isospin space results in additional couplings between the photon and intermediate vector bosons. This gives rise to a photon pseudo-mass which depends on the polarization state of the photon; hence an homilia string network acts like a birefringent medium. Isospin rotation of the Higgs field can be differentiated from quantum effects, since the former is coherent over a length scale of the order of the observable Universe. However, there is a large discrepancy between the predicted birefringent length scale and the observed length scale reported in reference [13]. It will require further investigation in order to determine whether the observed birefringence scale can be reconciled within the standard electroweak model.

ACKNOWLEDGMENTS

One of the authors (M.J.T.) acknowledges the support of the APA.

APPENDIX A: GAUGE INVARIANT EIGENVECTORS

The vector boson eigenvectors (18) are valid for a particular choice of local gauge in which the Higgs field is real. If the local gauge is not fixed the particle eigenvectors become

$$\mathbf{A}^\gamma(x) = \frac{\sin \theta_W}{\tan \theta_W |\Phi|^2} \begin{pmatrix} -|\Phi|^2 \\ \chi^1(x) \\ \chi^2(x) \\ \chi^3(x) \end{pmatrix} \quad (\text{A1a})$$

$$\mathbf{A}^Z(x) = \frac{\cos \theta_W}{\tan \theta_W |\Phi|^2} \begin{pmatrix} |\Phi|^2 \tan^2 \theta_W \\ \chi^1(x) \\ \chi^2(x) \\ \chi^3(x) \end{pmatrix} \quad (\text{A1b})$$

$$\mathcal{N}_\mu(x) \mathbf{A}^W(x) = \begin{pmatrix} 0 \\ \mathcal{A}_\mu(x) \chi^2(x) + \mathcal{B}_\mu(x) \chi^3(x) \\ -\mathcal{A}_\mu(x) \chi^1(x) + \mathcal{C}_\mu(x) \chi^3(x) \\ -\mathcal{B}_\mu(x) \chi^1(x) - \mathcal{C}_\mu(x) \chi^2(x) \end{pmatrix} \quad (\text{A1c})$$

$$\mathcal{N}_\mu(x) \mathbf{A}^{\overline{W}} = i |\Phi|^2 \tan \theta_W \begin{pmatrix} 0 \\ -\mathcal{C}_\mu(x) \\ \mathcal{B}_\mu(x) \\ -\mathcal{A}_\mu(x) \end{pmatrix}, \quad (\text{A1d})$$

where

$$\mathcal{A}_\mu = \frac{\chi^2 \partial_\mu \chi^1 - \chi^1 \partial_\mu \chi^2}{|\Phi|^4 \tan^2 \theta_W} \quad (\text{A2a})$$

$$\mathcal{B}_\mu = \frac{\chi^3 \partial_\mu \chi^1 - \chi^1 \partial_\mu \chi^3}{|\Phi|^4 \tan^2 \theta_W} \quad (\text{A2b})$$

$$\mathcal{C}_\mu = \frac{\chi^3 \partial_\mu \chi^2 - \chi^2 \partial_\mu \chi^3}{|\Phi|^4 \tan^2 \theta_W}. \quad (\text{A2c})$$

The normalisation constant \mathcal{N}_μ is

$$\mathcal{N}_\mu = \frac{\sqrt{(\partial^\mu \chi^a)(\partial_\mu \chi_a)}}{|\Phi|^2 \tan \theta_W}, \quad (\text{A3})$$

where the summation convention does not apply to μ in Eq. (A3). Note the eigenvectors for W and \overline{W} (A1c and A1d) cannot be expressed independently of the vector coupling fields \mathcal{A}_μ , \mathcal{B}_μ and \mathcal{C}_μ .

From the definitions of the vector boson eigenvectors in Eqs. (A1) we derive the relations

$$(\sin \theta_W)^{-1} \partial_\mu \mathbf{A}^\gamma = (\cos \theta_W)^{-1} \partial_\mu \mathbf{A}^Z = \mathcal{N}_\nu \mathbf{A}^W \quad (\text{A4a})$$

$$\partial_\mu \mathbf{A}^W = \mathcal{N}_\nu (\mathbf{A}^\gamma \sin \theta_W + \mathbf{A}^Z \cos \theta_W) + \tau_\mu \mathbf{A}^{\overline{W}} \quad (\text{A4b})$$

$$\partial_\mu \mathbf{A}^{\overline{W}} = -\tau_\mu \mathbf{A}^W. \quad (\text{A4c})$$

Comparing Eqs. (A4) with Eqs. (34) shows that the coupling strength ρ_μ is given by

$$\rho_\mu \equiv \mathcal{N}_\mu = \frac{\sqrt{(\partial_\mu \chi^a)(\partial^\mu \chi_a)}}{|\Phi|^2 \tan \theta_W}, \quad (\text{A5})$$

where $a = 1, 2$ and 3 , and the summation convention does not apply to μ . Using Eq. (A5), $\rho_\mu \rho^\mu$ is of the form

$$\rho_\mu \rho^\mu = \frac{(\partial_\mu \chi^a)(\partial^\mu \chi_a)}{|\Phi|^4 \tan^2 \theta_W}. \quad (\text{A6})$$

τ_μ is defined as

$$N_\alpha N_\beta \tau_\delta = \frac{\epsilon_{abc} \chi^a (\partial_\alpha \chi^b) \partial_\beta \partial_\delta \chi^c}{|\Phi|^2 \tan \theta_W}, \quad (\text{A7})$$

where ϵ_{abc} is the fully asymmetric tensor and we sum over α and β . In Eqs. (26) $\tau_\mu = 0$ due to the choice of gauge which resulted in $\chi^2 = 0$. We can always perform a gauge transformation such that $\tau_\mu = 0$ and hence τ_μ does not described a physical coupling. Employing the relations in Eqs. (A4) leads to the field strength $Tr(F_{\mu\nu} F^{\mu\nu})$ being described by Eq. (22), with ρ_μ defined by Eq. (A5).

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